Trading in the Brazilian Electricity Market, Capturing and Organizing Forward Price Curves and analyzing their Empirical Characteristics.

H. Leme, P. M. Hansen, L. K. Hotta, M. Zevallos

Abstract— **Trading in the Brazilian Electricity Free Market has several particularities not found in other mature markets, making risk assessment a difficult task, particularly when capturing forward price curve due to the lack of public price references and the information asymmetry among players. The forward price curve represents the price at which open energy positions at different maturities can be closed at a specific moment. This paper analyses the main trading characteristics of the Brazilian Electricity Market and proposes three ways to capture forward energy prices, how to organize that information and analyze it. Additionally, we study the empirical characteristics of a set of curves providing the essential foundation for modeling and understanding such series and its volatilities. We note that these curves have weak time dependency and volatility blocks dissipate quickly. We also note that relative monthly changes of the prices in the shorter maturity of the curve are approximately five times larger than those in the longer part of the curve, indicating that market risk declines when maturity increases.**

***Index Terms*—Forward curve, Brazilian electricity market, energy trading, energy prices.**

I. INTRODUCTION

The Brazilian Electricity Market (SEB) has several regulatory characteristics and particularities that expose players to risks factors not common in financial or other power markets.

In this market the vast majority of transactions are done in the OTC Market and transactions are not standardized, containing in general several embedded derivatives which complicate the pricing process and limit the discovery of market prices, as prices of these products cannot be compared directly.

Additionally, the only official price reference is the spot price (PLD) which is used to settle all deficits and surpluses in the free and regulated market, assess penalties and as a price reference for OTC contracts. However, PLD has a limitation because this spot price is the outcome of a sequence of computer models called NEWAVE and DECOMP [1] which only consider information on how to operate the SEB and not from real market information.

Recently, some organized markets have been created but so far, liquidity issues limit the representativeness of these exchanges. Moreover, the power purchase auctions for the Regulated Market (ACR) do not reflect appropriately the prices that are actually negotiated in Free Market (ACL), because the characteristics of each transaction are quite different, representing different risks for sellers. Lack of transparency and information asymmetry are characteristics of the OTC market in its initial stage. This complicates the process of pricing and risk measurement of energy operations.

Other factors such as climatic variables, especially wind and rainfall, economic performance, system generation expansion and regulatory rules also influence market prices in different time horizons.

These characteristics emphasize the need to build tropicalized models to attend the specific needs of the electricity market. Accordingly, the first step is to identify the main rules and characteristics of trading in this energy market and understand the risk factors involved in this process. This is necessary to define properly what the Electricity Price Forward Curve (EFC) represents and how to capture this information directly from the market. Based on the forward curve, it is possible to understand the term structure of the market prices and to assess the effect of potentially important variables in the forecasting process and volatility estimation of futures prices. Price volatility constitutes vital information for pricing and risk assessment models.

Most market agents have their own individual forward curve, which is estimated considering a number of information sources, including available quotes in the market, agents expectations and operations conditions over different time periods. These individual curves rarely reflect a complete overview of the market and can be strongly influenced by the capturing process adopted by each company and the specific energy position of a particular agent. Despite of that, they serve as good starting reference for market prices.

Although the Forward curves are critical for trading, they have received little coverage in the literature because there is no public information available to study theses quantities.

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This paper proposes an adequate structure to capture and organize forward curves in the SEB and shows the main empirical features of this price curve.

The remainder of the paper is divided as follows. In section II we describe how the SEB works, specially the main issues involved in energy trading. In section III we define the concept of the forward curve and suggest three approaches to capturing and organizing this information. In section IV we analyze in detail the curves in order to map their main characteristics. In Section V we present our conclusions and suggestions for future work.

II. Trading Electricity In Brazil

Generators, authorized traders (comercializadores) and free consumers (those with energy demand greater than 3 MW and supply voltage greater than 69kV) participating in the ACL are responsible for contracting their energy needs. They are free to negotiate prices and contracting terms and conditions directly with their counterparts.

In this environment the free consumers need to contract their energy consumption with enough flexibility to fit their consumption process. They need to consider economic performance and its potential effect on their production as they are not allowed to resell energy directly to the market. Suppliers generally structure customized products for their clients incorporating the volume risk into their own portfolios.

Although the ACL is a free environment, there are a number of rules and procedures, instrumented through the Electricity Clearing Chamber (CCEE), which effectively define how an agent should operate. The CCEE is responsible for the settlement of the ACR and ACL contracts, to calculate and publish the PLD, register and measure consumption and generation, control compulsory guarantee deposits and generate information.

Some rules are important and directly affect the way the market operates. First, all consumers must acquire and register physical energy contracts for 100% of their consumption. The physical backing has a vital role to protect the system from a systemic deficit and inhibit perverse speculation. Marketers and generators also need to prove their physical backing at settlement. To reduce default risk, all agents must deposit guarantees at the CCEE.

Physical backing is checked monthly based on a 12 month moving average of energy credits and debits at the CCEE. The credits are given by the quantities actually generated, registered purchase contracts or a certified guarantee given to a generator by the Regulatory Agency (ANEEL). Debits are granted to consumers when energy is consumed from the grid or generators or marketers when they are the seller part at a registered contract.

If an agent does not have sufficient physical backing at a given month, in addition to paying the PLD price for the energy in deficit, he receives a penalty given by one twelfth of the highest value between PLD and a Reference Value (VR) specified by ANEEL (National Agency of electricity) for each year, multiplied by the amount of energy in deficit. This penalty can be severe, considering that VR for 2012 is R$ 141.72/MWh and that PLD can range from R$ 12.20/MWh to R$ 727.52/MWh depending on the SEB conditions.

To encourage the construction of alternative generation sources of energy in the grid, the government created incentives via discount in the distribution system rate (TUSD) for those consumers who purchase this energy type. These are typically biomass cogeneration plants, small Hydro Power Plants (PCH) or wind farms. Consumers pay a higher price for this energy as they will be compensated by the reduction in the TUSD rate. The discount depends on the Distribution concession area where the consumer is located, which makes this kind of subsidized energy more attractive in certain regions of Brazil.

Conventional energy is that generated by large hydro plants or generated using fossil fuels, or those sources of biomass or wind that were not certified for discount in TUSD.

There is also a type of energy with 100% discount on the TUSD rate, which goes through a differentiated certification process.

The CCEE controls the physical backing for each energy type where conventional energy cannot be used to cover the potential deficit of subsidized energy. Basically, there are three types of energy (conventional, I50% and I100%), each with its specific rules for accounting physical backing. In practice, the differential treatment by energy type substantially increase the risk agents have when managing their portfolios, because an agent can have a balanced position of energy but can be penalized for imbalances between energy types.

Because there is very little available information about price curves for I50% and I100% energy and computer models NEWAVE and DECOMP only apply for conventional energy forecasting prices for different energy types is one of the main challenges of operating in the ACL.

Moreover, the risk profile and variables associated with the decision making process for each energy type is very different, adding complexity to the contracting process and energy portfolio management.

In fact, the only official information about market prices is the PLD. To calculate the PLD, a chain of computational models, called NEWAVE DECOMP are fed with lots of information about the SEB structural conditions and expected rainfall and projected growth in energy consumption. This combination of data, called DECK, maps operating variables and characteristics of supply and demand for energy. The DECK is the input of a dual stochastic optimization process that is performed by software NEWAVE and DECOMP and the main output of this process is the marginal cost of operating the system (CMO) which is then used by the CCEE to calculate the PLD. For details of the mathematical formulation of these models and its implications see [1].

There are many criticisms to the approach used in determining the spot price; see [2]. However, the positive side to the NEWAVE model is that agents may alter its inputs in the DECK to reflect their own expectations. For example, they can measure the price effect of delays in expansion premises, measure sensitivity to energy consumption changes and incorporate climate effects such as El Niño and La Niña.

Since the PLD is used as the settlement price for the CCEE, it becomes the main reference for short term trading. Many contracts are PLD plus a premium, also denominated locally as *Spread*. The *Spread* varies according to market conditions like liquidity and credit risk. Often the value of the spread is greater than the PLD itself.

An important issue when pricing energy contracts is related to the real underlying product that should be modeled at each time interval of the contract. NEWAVE can create various scenarios for future PLDs but because of its high sensitivity to certain inputs, like the hydrological scenario, the volatility of these prices could be very high representing unrealistically the underlying product for an interval further than 4 to 6 months [3].

As a result, the PLD used to assess long-term contracts overestimates the risk and the value of embedded derivatives in energy contracts.

Therefore, it is important to have an alternative way to monitor long-term market prices and measure its volatility realistically.

The forward price curve concept is suitable for the SEB, as it captures many of the important market features because the Forward curve is based on real prices being traded and reflects the player's expectations considering what price they can mark to market their open positions [4].

Thus, the Forward curve is associated with a market vision of energy prices, which incorporates risk factor premiums, opportunity cost, liquidity, market concentration, and also allows calculate price volatility realistically, because prices reflect a market behavior and not an operational characteristic of the system.

III. Energy Forward Curve

A. Definition and characteristics

Let $F_{t,h}$ represents the price of energy defined at *t* for a maturity *h* where *t* is the time when the price is measured and *h* is the maturity to the supply of energy. Then a Forward curve in t is $F_{t,h}$ calculated for different values of h, that is, energy prices for several maturities, measured in t.

In the SEB, the Forward curve reflects the fixed price of a Swap for a certain supply interval, generally monthly or annual. Thus, the interpretation of a Forward curve is not that of a future contracts traded on an exchange, which is defined in a single future point in time.

We can obtain Forward curves for different energy types and for each submarket in the SEB, where PLDs are calculated separately.

B. Capturing

The Forward curve should reflect the price at which any agent at time t, could open or close an energy position at a certain maturity h. An exchange Forward curve is calculated with effectively closed deals and reflects the intersection of supply and demand. In markets where exchanges are illiquid Forward curves could be measured based on market price estimations.

Another possibility is to use player's individual estimations as reference for the Forward curve. However, these individual estimations can vary not only by market information asymmetry, but also due to the risk premium or opportunity cost used by each agent.

To capture this information directly from the market we suggest three approaches, which can be applied depending on the available information each player has. Notably, players with better information would have more realistic estimates of price curves than those who only have access to public information.

C. Individual curves based on market interaction

The first alternative is for each player to use their own individual forward curve. Many already have an internal process to capture and store this information and despite its possible limitations, it can still be used to understand price dynamics and for product pricing and risk assessment.

In particular, most companies do not have a quantitative mechanism to build and use these curves and still use the trader sentiment as the market prices reference. Bias and perspective can have a large impact in these cases, and we expect the most active players to have a better quality curve.

When constructing an individual Forward curve we assume that the player has a reliable and large enough network of contacts he can consult with, which is not the case in many companies, and a competent internal analytical team that can complement with analyzing the fundamentals of the market. However, many industry players do not meet these requisites because they have no active marketing or well informed support team.

D. Pool of individual curves

The second alternative is to capture using a representative group of market players (Pool), as many realistic forward curves as possible. With that information organized by data capture, maturity and for each energy type, we can define statistically representative metrics that reflect the electricity forward curve and its dynamics. This could be the ideal case as each individual curve from the Pool captures both information of real transactions being closed in the OTC and the expectations of each player. Given the characteristics of trading in this free market and its liquidity, the Pool can realistically capture monthly prices for up to three months maturity and yearly prices for the next year onwards for up to five years ahead, both for conventional and I50% energy type.

Before calculating the metrics from the observations in the Pool, a statistical treatment should be applied to the data to avoid undesirable effects such as the impact of outliers, which could occur by players trying to change artificially the metrics or agents outside of consensus

Standardizing Information has many advantages; it allows us to compare the curves over time and measure the volatility associated with each product.

Each participant of the Pool would be benchmarking their individual curve using those metrics he considers most representative. This allows them to have a broader view of market prices, because the information is generated by a representative and aggregate group of market players.

This concept may looks simple, but the implementation of this structure is essentially complex. Dcide, a company specializing in data processing and risk analysis, implemented and operates a Pool of these characteristics since January 2012.

Players are willing to input their individual Forward curves to the Pool as long as the information is kept secure, used exclusively to calculate the metrics and not revealed to any third party. Ensuring players input values that reflect their true vision on market prices, organize this information into a logical structure, ensuring transparency and absolute confidentiality of participants individual information, treating distortions caused by intent to vitiate the general metric and ensure long-term sustainability of the Pool are the most challenging tasks of this approach.

Forward Curves built from a Pool of individual curves have several desirable features. They reflect a broader picture of the market, eliminating the bias any individual player may have, and can be evaluated for various standard products and time periods. Therefore it can be used as a reference to mark to market open positions and used as a reliable benchmark proxy to price products with embedded flexibilities.

E. NEWAVE outputs as Forward price proxy

The third alternative is to try to infer market prices based on publicly available information. The problem with this approach is that the data reflects an official vision on how to operate the system, with little or no information about actual trading being done [1]. An alternative approach is to try to relate the prices of the individual Forward curve to the PLD or other public information that could influence prices and, based on this relationship, build a forward price curve using only public information. These models should be periodically reassessed, since the structure of the energy market is dynamic.

The next step is to organize the curves logically in order to study their characteristics in time. We suggest a methodology in the next subsection.

F. Organization

The Forward curves analyses are related with the **frequency** which the curve is captured (monthly, weekly, etc.) and the distance to the delivery (**maturity**) of the underlying commodity.

Forward curves can be structured as a set of time series for each product quoted by the agent, where the product is defined by type of energy and maturity. It is common naming these curves according to its product maturity. We adopt the following naming rule: M +1, M +2 and M +3 for products with monthly supply and maturity equal to 1, 2 and 3 months ahead, respectively, A +0 for the product that reflects the supply from the fourth month ahead to the end of that year, and A + n, n = 1,2, .. products of annual supply maturing in n years. Table 1 is an example of organized curves from A+0 to A+5.

Some players who don't have individual curves have to take decisions based on public information, particularly the PLD.

Each run of the NEWAVE model gives us 2000 CMO simulations for each month in a five year horizon called the planning horizon [1].

TABLE I
INDIVIDUAL FORWARD CURVES FOR A TO A+5 MATURITIES, MEASURED FROM AUGUST 2008 TO FEBRUARY 2009.

Curva Forward	Ano A+0	Ano A+1	Ano A+2	Ano A+3	Ano A+4	Ano A+5
...						
08/2008	160	190	180	170	160	125
09/2008	155	170	175	170	160	125
10/2008	115	160	150	140	140	125
11/2008	115	140	140	145	135	125
12/2008	110	130	130	135	135	125
01/2009	120	130	135	135	125	125
02/2009	135	133	133	133	130	130
...						

The CMO is somehow associated with the PLD and therefore we can use some of these simulations as a proxy for the PLD. The common approach is to truncate each of the 2000 simulated series of CMO with the minimum and the maximum PLD limit, and calculate the monthly average of these quantities. These series can be organized in the form M +1, M +2, M +3 +0 A, A +1, ..., A n + and used as an approximation of the forward curve. We will call forward PLD curves those formed by the NEWAVE M +1, M +2, M +3 +0 A, A +1, ..., A+n values for each DECK.

For practical purposes, CCEE configuration DECK could be used to calculate a monthly forward PLD.

We study the dynamics of forward curves for various maturities to understand their relationship. Particularly, it is important to map key variables that could justify the price dynamics in different time horizons and explain its structure through a time series model.

G. Volatility

The volatility is the main input to calculate risk of energy contracts or portfolios, price contractual flexibilities and mark to market open positions.

The quality of the volatility estimations is of upmost importance because it influences directly on the premium for flexibilities embedded in energy contracts.

One way to define volatility is as the standard deviation of the relative price changes in the forward curve, for each time scale of measurement. Volatility is commonly presented in a relative manner for the underlying product and you can convert volatility to different time scales.

Let $F_{t,A+n}$ the price of the forward curve for a A+n maturity, measured at *t* moment in time, then

$$x_{t,A+n} = \frac{F_{t,A+n} - F_{t-1,A+n}}{F_{t-1,A+n}} \quad (1)$$

is the relative price change, delta, from the moment *t*-1 to *t*. The volatility at time *t* is defined as

$$\sigma_{t,A+n} = \sqrt{\mathrm{E}_{t-1}[x^2_{t,A+n}] - \mathrm{E}_{t-1}{}'[x_{t,A+n}]^2} \quad (2)$$

where $\mathrm{E}_{t-1}[.] = \mathrm{E}[.\,|\,\Im_{t-1}]$ is the expectation conditioned to all information observed up to time *t*-1, $\Im_{t-1}$. Equation (2) suggests that volatility has a dynamic structure. Note that knowing the conditional probability distribution allows us to know the volatility.

In the next section we will study in greater detail the characteristics of the Forward curve from a term structure and volatility standpoint.

IV. Empirical Analysis of the Forward Curves

The forward curves used in this analysis correspond to a major trading firm in Brazil which has one of the biggest market shares. For confidentiality reasons we will not reveal the name. We captured the information monthly.

Each captured curve represents the best estimated price for delivery of conventional energy in the southeast (SE) submarket for each year up to five years ahead. The data were sent regularly from the front office to the middle office via e-mail and a validation process was done to verify the quality of information. This information is deemed official within the company and used as a reference to analyze its portfolio of contracts, calculate risk, mark to market and price flexibilities.

The dataset considered here is composed of 88 price curves quoted monthly, from January 2005 until April 2012. Altogether there are 6 sets of price series for maturities A+0 until A +5. No special treatment was given to the observations. Figure 1 shows the described forward curves.

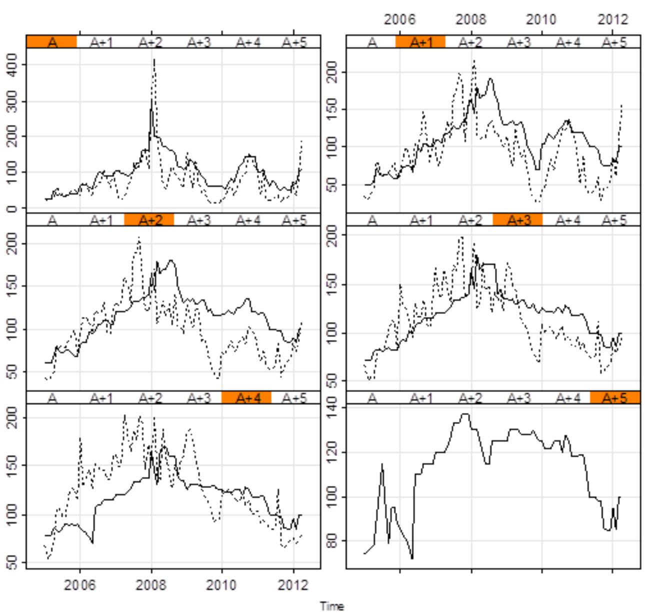

Fig. 1. Forward curves (full lines) and forward PLD curves (dotted lines) for A+0 to A+5 curves from January 2005 to April 2012.

A. Forward Curve and PLD

The discussion carried out in sections I and II suggest that the Forward's curves are associated with factors that influence its dynamics in the short and long term horizons. In the short term, these curves tend to have higher variation and are associated with factors such as climate and operational issues. These factors also affect the PLD, then a first question is whether the forward curves are associated with the forward PLD curve. The forward PLD curve is preferred to the performed PLD because theoretically, this amount reflects a projection for the same time horizon we have in the forward curve. In Figure 1 we present for each forward curve maturity A+0 to A+5 the corresponding forward PLD curve at the same measurement time.

Note that there is some adherence between the forward curve and the forward PLD curve for maturities A+0 to A+4..

This characteristic was expected because the prices of short-term energy and their contracting decisions are influenced by the PLD, because it is the only available public price reference.

When the time horizon increases, current system conditions have secondary impact and additional others factors such as system generation expansion, market growth which depends on macroeconomic performance and the risk premium required by market participants begins to dominate expectations, and as the forward PLD curve does not reflect adequately these issues, the adherence tends to decrease.

B. Temporal Dynamics

From Figure 1 we observe that Forward curves reached their highest values in the first quarter of 2008, caused mainly by structural gas supply uncertainties that affected both the PLD calculations and the player's expectations. After that and in line with the 2008 international crisis, the drop in prices was explained by a larger supply growth compared to demand.

To analyze the dependence structure of the level we can work with the delta series, $\{x_{t,A+n}\}$, because the originals series are nonstationary. The deltas series for A+5 to A+0 from January 2005 to April 2012 are presented in Figure 2. First we note that the range of variation of curves A+0 and A+1 outweigh those of the longer maturity, indicating that the volatility in the shorter-term maturities is higher than the longer-term. The figure also indicates there is no trend or seasonality.

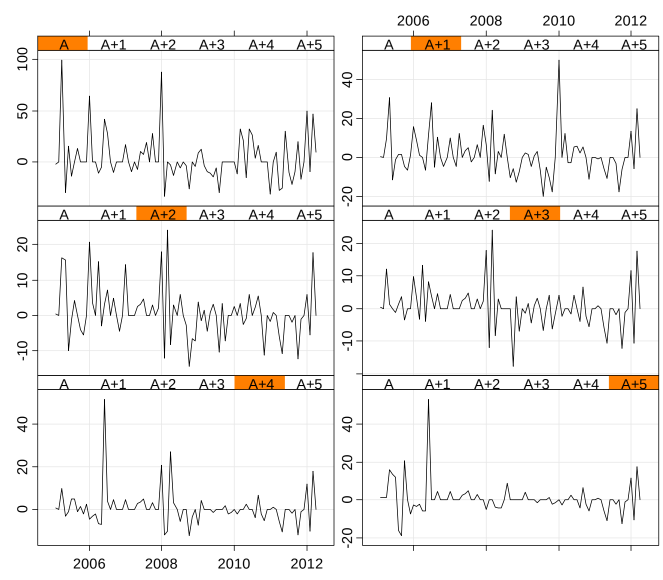

Fig. 2. Monthly relative deltas (%) for A+0 to A+5 curves from January 2005 to April 2012

When analyzing the autocorrelation function of $\{x_{t,A+n}\}$ for the series A+0 to A+5, shown in Figure 3, we don't see a strong correlation, indicating that the temporal dependence structure at the level of the series can be approximated by a low-order autoregressive or white noise models. In this case, as the temporal dependency of $\{x_{t,A+n}\}$ does not seem to be significant, we can study series functions autocorrelations without filtering the data.

In Figure 4 we present the autocorrelation functions for the squares of the deltas, $\{x^2_{t,A+n}\}$. The figure suggests that the values are not significant indicating that the volatility has no persistence, and that the effect of extreme events dissipates quickly. In fact, many of the events that alter energy prices occur within a given month, and don't affect the market for long periods of time. Even in times of high price uncertainty, generally associated with crises that impact energy consumption or the transition from wet and dry seasons, price volatility returns to historical levels quickly.

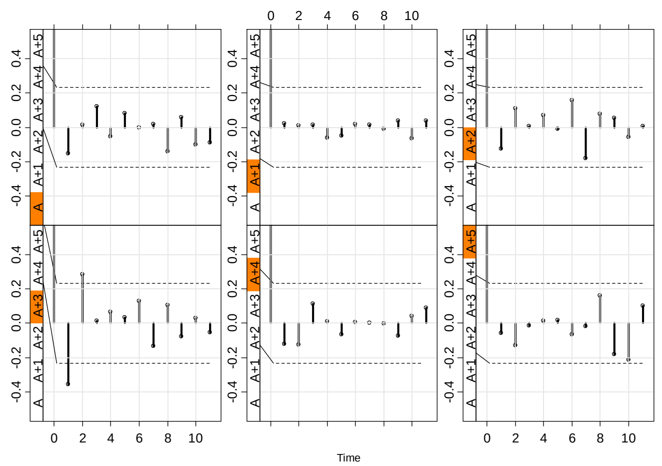

Fig. 3. Autocorrelation function of monthly relative deltas for A+0 to A+5 curves. Dotted lines show a 95% confidence interval.

C. Volatility

Volatility is defined by the equation (2) and is a function of the square of the relative forward curve deltas. Thus, we can study the magnitude of volatilities analyzing the delta absolute values probability distributions for each maturity. The delta absolute values are shown in Figure 5, and the histogram of these quantities in figure 6. Note that the frequencies in the histogram are more concentrated in the zero neighborhoods for longer maturities, indicating that variability decays for longer maturities, as is widely known. Additionally, there seems to be some periods where there is more variation than others, which could indicate blocks of volatility.

Note also that Figure 6 shows several peaks for all maturities. In the A+0 series the delta absolute value reached 100%, indicating that in some periods the price doubled in value. In this sense, the peaks for maturities A+4 and A+5 that occurred in 2006 are atypical when compared with other observations and can significantly affect estimates of potential models for volatility. With regard to seasonality, we noticed no cyclical pattern, and therefore the absence of this effect.

The histograms shown in Figure 6 indicate more concentration around zero value for longer maturities in line with the behavior observed in Figure 5, which presents absolute deltas with variation decreasing according to maturity.

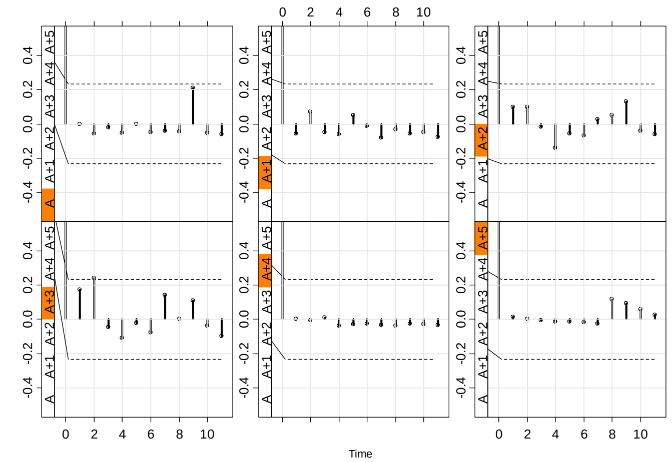

Fig. 4. Autocorrelation function of the squares of monthly deltas for A+0 to A+5 curves. Dotted lines show a 95% confidence interval.

The peaks in the histogram frequencies occur at point 0 which is the result of numerous repetitions of curves values in subsequent observations. These repetitions are justified by a rounding effect, as historical values of the curves were filled without considering the decimal place, so changes lower than $1.00 are not captured by the curves. In the long part of the curve, this effect is more pronounced, since the curves tend to have fewer changes. Additionally, as there is less liquidity for the longer maturity products the trader may take longer to perceive a price change.

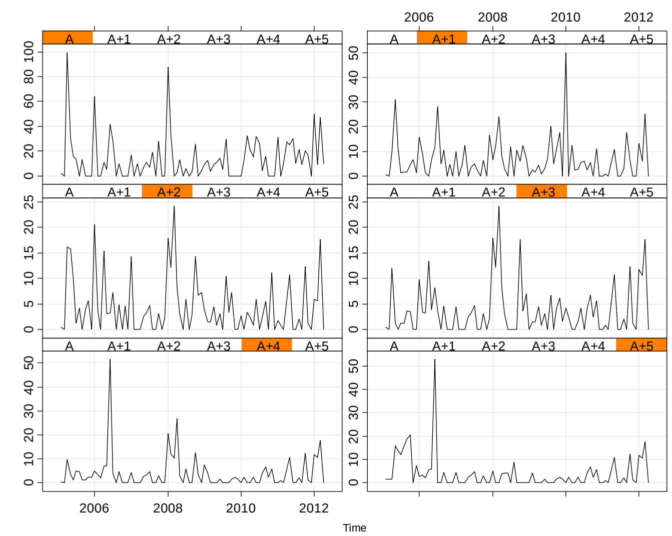

Fig. 5. Monthly deltas mode (%) for A+0 to A+5 forward price curves.

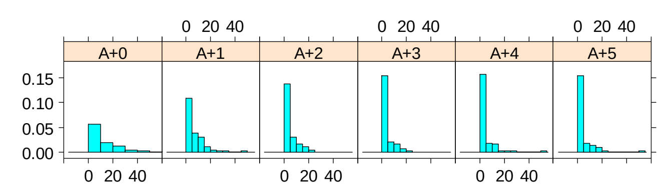

Fig. 6. Deltas (%) absolute value histogram for A+0 to A+5 curves. The histogram rage is from 0 to 60 implying that the three higher values are not showed for the A+0 curves.

The deltas absolute values medians are showed in Table 2 presents for each curve.

TABLE II
MEDIANS FROM THE DELTAS ABSOLUTE VALUES FOR A+0 TO A+5 CURVES FROM 2005 TO 2012.

	Ano A+0	Ano A+1	Ano A+2	Ano A+3	Ano A+4	Ano A+5
Median	9.1%	4.5%	2.6%	1.6%	1.6%	1.5%

The medians confirm that the volatility in the first three maturities is higher than in the rest. From the median perspective, the deltas in A+0 is twice as big as in A+1 and approximately five times greater than in the A+5. Medians do not incorporate any information about the duration of the low and high delta periods, but one way to capture this information is to count the number of periods subsequent to the absolute variations are above and below its unconditional median. This analysis is presented in Figure 7 for curves A+0 to A+5.

Values "above" indicate that the variation was greater than the unconditional absolute median in that month, values "under" indicate that this variation was lower than the median. Thus it is possible to measure the size of periods of high and low volatility. The numbers in the panels represent the sizes of the volatility blocks, which are defined as the number of subsequent observations above or under the unconditional absolute median.

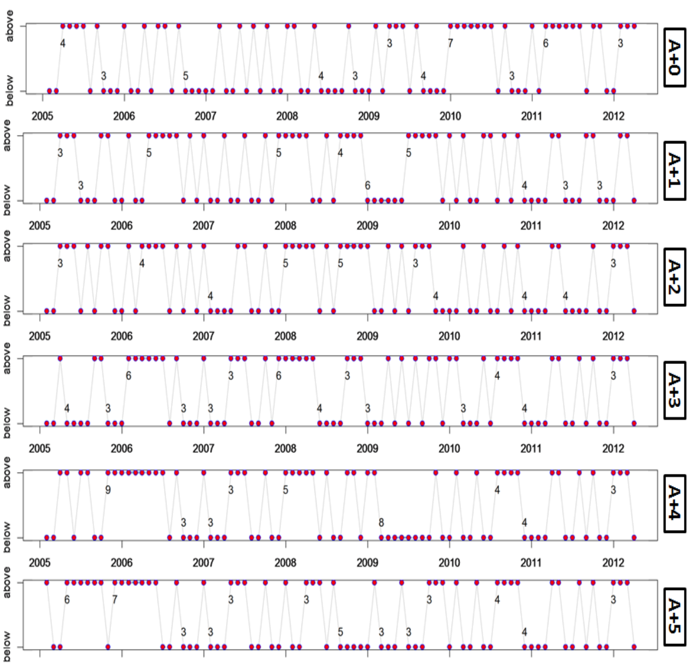

Fig. 7. Volatility blocks duration. Above and below median for monthly deltas for A+0 to A+5 curves.

Figure 7 suggests that the size of the blocks is relatively small, 3-9 observations indicating that there is little persistence. Hypothesis testing (Runs-test) were performed to verify if the independence assumption is reasonable for the data of each individual panel associated with each of the forward curves. The results of the test-runs indicate that the assumption of independence cannot be rejected (p-value > 0.05 for all curves), so the behavior observed in blocks could be generated by a white noise.

It is not surprising to observe low persistence in the absolute returns values if we consider that the data were collected on a monthly basis.

It is important to note that volatility, according to equation (2), is not the same as the square or modulus of monthly variations. Volatility is the conditional standard deviation, which can be modeled as a function of the absolute values or squares of monthly variations. Thus, the model and the estimator chosen determine the way absolute values of the latest variations affect the volatility.

D. Term Structure

Although the time dependence structure of deltas in monthly forward curves seems to be weak as seen in Figure 3, the term structure of forward curves, as a function of maturity, could be relevant. In Figure 8 we show a surface representing the curves as a function of time and of maturity.

Note that these curves move similarly in time, although the curves A+0 and A+1 show greater variation, with clear peaks and valleys. This can be justified because short-term factors drive the first two years.

From a maturity perspective, the relationship between these curves seems to change in time according to the structure dynamic of the series in time.

One way when modeling the term structure of the forward curves is using the Diebold-Li approach [8], or its generalizations. These models use a transformation called forward rate instead of the original forward curve, and can be calculated using the following relationship $F_{t,A+n} = PLD_t \cdot e^{r_{t,A+n} \cdot 12(n+1)}$ wheree $F_{t,A+n}$ is the forward curve measured at t for a A+n maturity, PLD_t is the average PLD for the month t and $r_{t,A+n}$ is the monthly forward rate for the month t at each maturity A+n, n=0,...,5. Thus, for each period t and forward term A+n, we haves

$$r_{t,A+n} = \ln\left(\frac{F_{t,A+n}}{PLD_t}\right) \cdot \frac{1}{12(n+1)} \qquad (3)$$

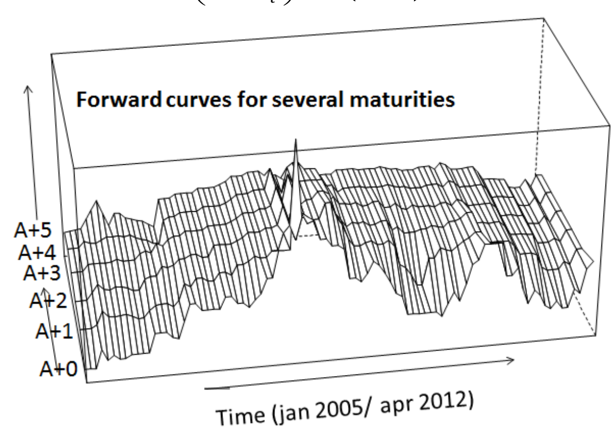

Fig. 8. Time evolution surface of forward curves for A+0 to A+5 maturities from January 2005 to April 2012.

The Diebold-Li model is a dynamic model with three latent factors for each month where we have quotes. The latent factors are associated with loads that reflect changes in the rate of forward prices [5]. These models attempt to capture the term structure of forward rates. One view of this structure are shown in Figure 9 for all forward curves. Note that the behavior of these curves is smooth in time and shows that the decrease with respect to maturity is exponential at times when rates are higher, while in the center of the data found a linear behavior.

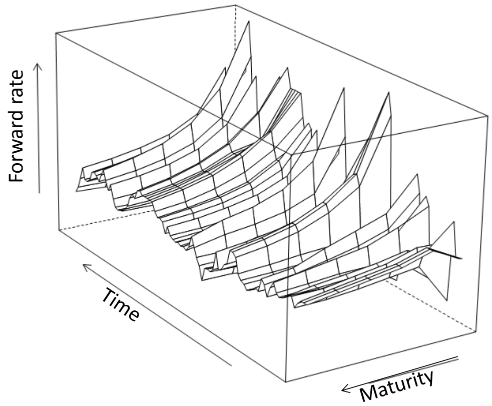

Fig. 9. Forward rates for A+0 to A+5 maturities from january 2005 to april 2012.

For the dataset considered in this study we adjusted the structure proposed by Diebold-Li and noted that the model was able to capture the term structure of forward rate, but did not produce good results in forecasting forward curve. Additionally, these models were not built to model volatilities as in equation (2).

V. Final Considerations and Future Development

This paper discusses the main issues related to trading electricity in the Brazilian free market, showing its main characteristics and risk factors. In this sense, the biggest challenge is to obtain accurate forward price curves, in order to have reliable input to pricing flexibilities, calculate market risk and mark-to-market open energy positions.

We presented three approaches to capture this information and suggested the Pool method as the most appropriate as it considers aggregate market information and not just that of an individual player.

As a starting point for many other future developments, we present an empirical analysis of a set of forward curves from one of the main market player, showing their main characteristics and providing the foundation for modeling these price curves.

We noted that the temporal dependency of the forward curve level is not significant and that delta blocks dissipate quickly, suggesting that volatility does not persist. We also found that the term structure of the curves evolves smoothly, but, the conversion of forward rates to forward price curves can be a problem.

The next steps would be to build forward curve models that can reflect the characteristics discussed in section IV in order to estimate volatilities realistically and obtain projection errors within tolerable levels.

Preliminary studies indicate volatilities for longer maturities (above two years) could be captured by ARCH model. For shorter maturities, estimate volatility based on a modified standard deviation that consider exponentially decreasing weights according to recent historical could be appropriate.

Additionally, we will monitor the consistency and quality of information gathered through the Pool to detect new behavior characteristics and other complementary information to improve the price discovery process in the Brazilian energy market.

VI. References

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VII. Biographies

Henrique Leme Felizatti received the B.S. and M.S., degrees in Statistics from University of Campinas (Unicamp), Campinas, Brazil, in 2005 and 2009 respectively. He was senior market risk analyst at CPFL Energia and co-founded Dcide Ltda. Currently he teaches Statistics in Goias Federal University and is Director at Dcide Ltda. His current research interests include market risk and portfolio management theory, energy derivatives, energy price and load modeling.

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